

HYDRAULIC TURBOMACHINES

Exercises 2 - Velocity Triangles

Parametric Study for a Velocity Triangle of a Francis Turbine

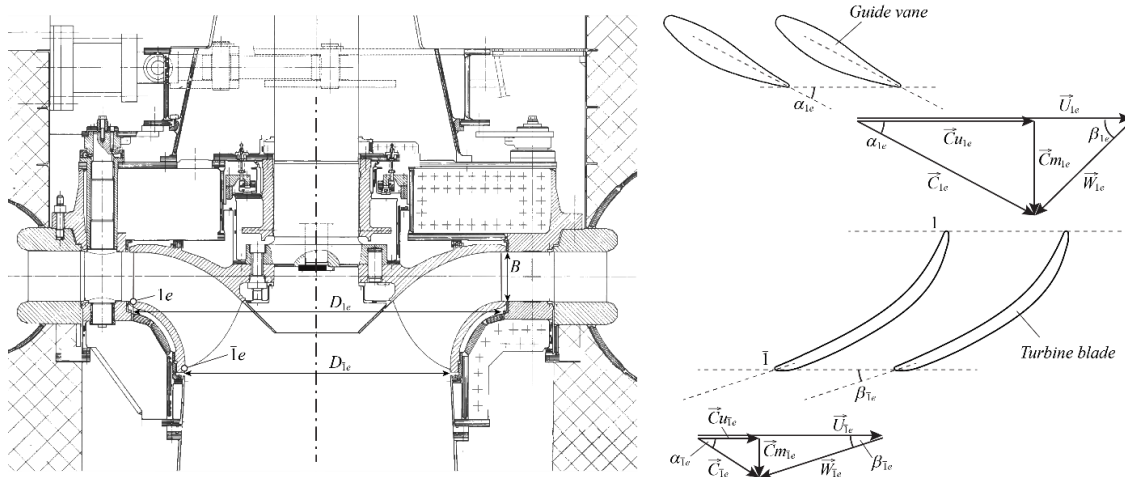


Figure 1. Scheme of the hydropower plant with its characteristics.

The meridional view of a Francis turbine and one example of the velocity triangle are sketched in Figure 1. For a Francis turbine, the angle of the absolute flow velocity at the inlet α_{1e} corresponds to the guide vane opening degree, and the angle of the relative flow velocity at the outlet corresponds to the outlet blade angle β_{1e} , as shown in Figure 1. Referring to the figure, answer the following questions.

- 1) Give the expression of turbine rotational velocity U_{1e} and U_{1e} as functions of the angular rotation ω and the inlet and outlet diameters, D_{1e} and D_{1e} respectively.
- 2) Give the relation of the turbine discharge Q and the discharge Q_t traversing the runner as a function of the volumetric efficiency η_v .
- 3) Give the meridional components of the flow velocity $C_{m_{1e}}$ and $C_{m_{1e}}$ as functions of the discharge Q and the volumetric efficiency η_v by using the geometrical parameters defined in the left part of Figure 1.
- 4) Considering the vectorial relationship at the turbine runner inlet 1, write the relation of π , $C_{u_{1e}}$, Q , η_v , D_{1e} , B and α_{1e} .
- 5) Considering the vectorial relationship at the turbine runner outlet $\bar{1}$, derive the relation of π , $C_{u_{1e}}$, U_{1e} , Q , η_v , D_{1e} and β_{1e} .
- 6) Derive the relation of the transformed specific energy E_t as a function of U_{1e} , U_{1e} , Q , η_v , D_{1e} , D_{1e} , B , α_{1e} and β_{1e} .

- 7) Considering the ratio of $\frac{U_{1e}}{U_{1e}^-}$, derive the relation for transformed specific energy E_t as a function of U_{1e}^- , Q , η_v , D_{1e} , B , α_{1e} and β_{1e} .
- 8) For a given rotational frequency of the runner, sketch the transformed specific energy E_t as a function of the traversing discharge Q , and derive the condition of minimum discharge Q_t^{min} to achieve positive specific energy.
- 9) When the turbine is operated at the best efficiency point (BEP), express the transformed power P_t with necessary variables, considering the assumption of the best efficiency point ($Cu_{1e} = 0$).

Calculation of the best efficiency using a hill-chart

The $Q_{ED} - n_{ED}$ hill-chart of a Francis turbine with the iso-value curves of both the global efficiency η (red curves) and guide vane opening α (blue curves) is represented in Figure 3. The horizontal and vertical axes represent IEC discharge factor Q_{ED} and IEC speed factor n_{ED} , respectively. Using the hill-chart, answer the following questions. Use the following values if required, where η_v and η_{me} are the volumetric and mechanical efficiency, respectively:

$$D_{1e} = 4.20 \text{ m}, D_{1e}^- = 3.50 \text{ m}, B = 0.60 \text{ m}, n = 3.88 \text{ Hz}, \eta_v = 0.98 \text{ and } \eta_{me} = 0.97$$

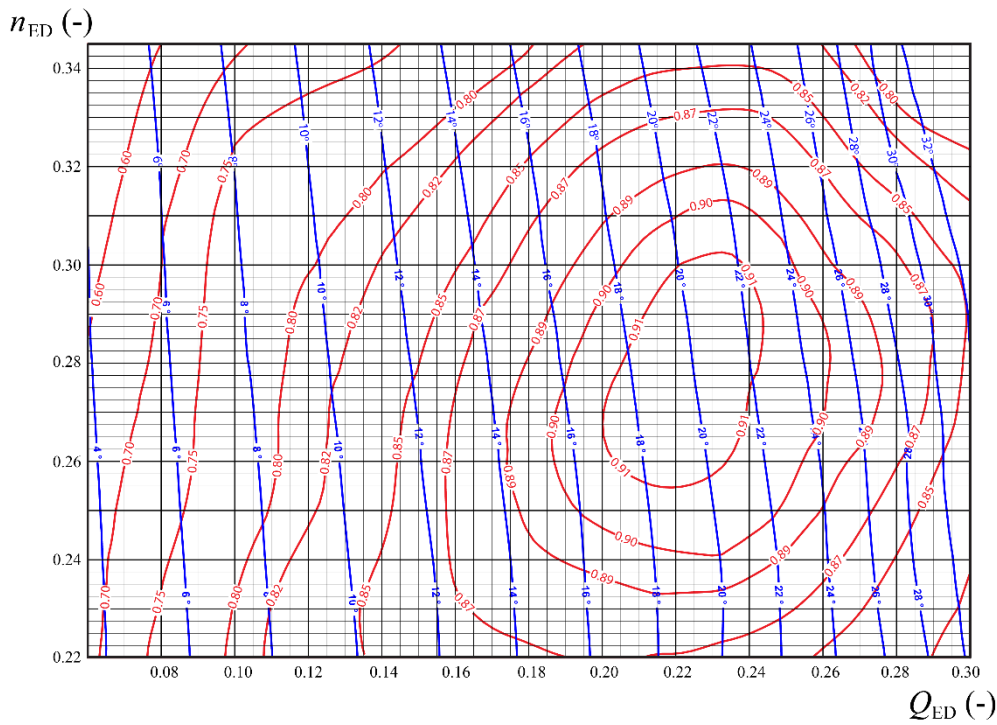


Figure 2. $Q_{ED} - n_{ED}$ hill-chart of a Francis turbine

- 10) Point out the best efficiency point in the hill-chart, and estimate the global efficiency $\eta_{BEP}^{hill-chart}$ and guide vane opening $\alpha_{BEP}^{hill-chart}$ at the best efficiency point (BEP).
- 11) At the BEP, the available head H and the discharge in the power plant Q are measured as $H = 235 \text{ m}$ and $Q = 130 \text{ m}^3 \text{ s}^{-1}$. Calculate the transformed energy E_t at the best efficiency point. Then, calculate the available power at the BEP, i.e. P^{BEP} .